Math 150 Test 5 Name: Key

Show all necessary steps Clearly, Neatly, and Systematically to receive full credit. Any incorrect statement will be penalized.

1. Solve: $2x^{\frac{2}{5}} + 3x^{\frac{1}{5}} = -1$.

Let $u = x^{\frac{1}{5}} = \sqrt[5]{x}$

$2u^2 + 3u = -1$

$2u^2 + 3u + 1 = 0$

$(2u+1)(u+1) = 0$

$2u + 1 = 0$	$u + 1 = 0$
$2\sqrt[5]{x} + 1 = 0$	$\sqrt[5]{x} + 1 = 0$
$\sqrt[5]{x} = -\frac{1}{2}$	$\sqrt[5]{x} = -1$
$(\sqrt[5]{x})^5 = (-\frac{1}{2})^5$	$(\sqrt[5]{x})^5 = (-1)^5$
$x = -\frac{1}{32}$	$x = -1$

$\{-\frac{1}{32}, -1\}$ //

2. Write the logarithm as the sum and/or difference of logarithms: $\log_3 \sqrt[5]{\frac{x^3y^2}{z^4}}$.

$= \log_3 \left(\frac{x^3y^2}{z^4}\right)^{\frac{1}{5}}$

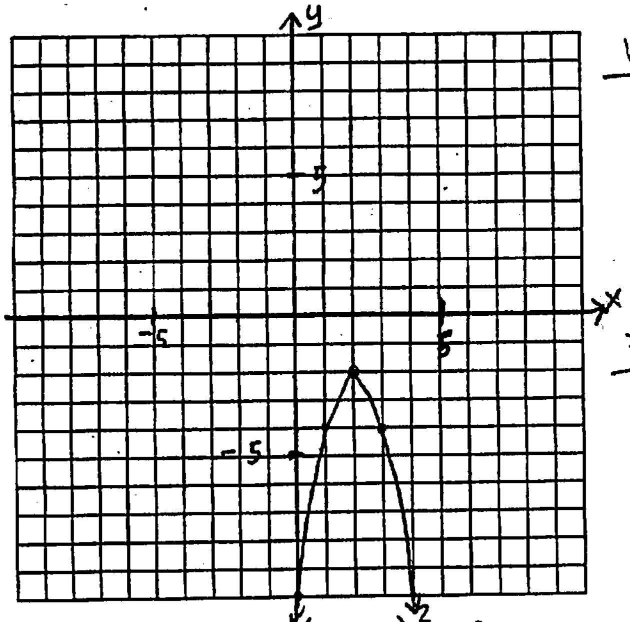
$= \frac{1}{5}\log_3\left(\frac{x^3y^2}{z^4}\right)$

$= \frac{1}{5}\left[\log_3(x^3y^2) - \log_3 z^4\right]$

$= \frac{1}{5}\left[\log_3 x^3 + \log_3 y^2 - \log_3 z^4\right]$

$= \frac{1}{5}\left[3\log_3 x + 2\log_3 y - 4\log_3 z\right]$ //

3. Let $f(x) = -2x^2 + 8x - 10$. (i) write $f(x) = a(x-h)^2 + k$, (ii) vertex, (iii) axis of symmetry, (iv) max or min function value, (v) x-intercept, (vi) y-intercept, (vii) sketch.



vertex

$$x = -\frac{b}{2a} = -\frac{8}{2(-2)} = 2$$

$$y = f(2) = -2(2)^2 + 8(2) - 10 = -2$$

x-intercept

$$0 = -2x^2 + 8x - 10$$

$$0 = x^2 - 4x + 5$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2} \leftarrow \text{complex number}$$

y-intercept

$$f(0) = -2(0)^2 + 8(0) - 10 = -10$$

(i) $f(x) = -2(x-2)^2 - 2$

(ii) $(2, -2)$

(iii) $x = 2$

(iv) max value $= -2$

(v) no x-intercept

(vi) $(0, -10)$

4. Write the logarithmic expression as one logarithm: $3\log_c(x+1) - 2\log_c(x+2) + \log_c x - \frac{1}{2}\log_c z$.

$$= \log_c (x+1)^3 - \log_c (x+2)^2 + \log_c x - \log_c z^{\frac{1}{2}}$$

$$= \log_c (x+1)^3 + \log_c x - \log_c (x+2)^2 - \log_c z^{\frac{1}{2}}$$

$$= \left[\log_c (x+1)^3 + \log_c x\right] - \left[\log_c (x+2)^2 + \log_c z^{\frac{1}{2}}\right]$$

$$= \log_c \left[(x+1)^3 x\right] - \log_c \left[(x+2)^2 z^{\frac{1}{2}}\right]$$

$$= \log_c \left(\frac{x(x+1)^3}{(x+2)^2 z^{\frac{1}{2}}}\right)$$

5. Let $f(x) = 2x+1$ and $g(x) = x^2 - 1$.

a. Find $(f \circ g)(-2)$.

$= f(g(-2))$

$= f(3)$

$= 7$ //

side

$g(-2) = (-2)^2 - 1$

$= 4 - 1$

$= 3$

$f(3) = 2(3) + 1$

$= 7$

b. Find $(f - g)(x)$.

$= f(x) - g(x)$

$= [2x+1] - [x^2 - 1]$

$= 2x + 1 - x^2 + 1$

$= -x^2 + 2x + 2$ //

c. Find $(g \circ f)(x)$.

$= g(f(x))$

$= g(2x+1)$

$= (2x+1)^2 - 1$

$= 4x^2 + 4x + 1 - 1$

$= 4x^2 + 4x$ //

d. Find domain of $\left(\frac{f}{g}\right)(x)$.

domain of $f(x)$: $(-\infty, \infty)$

domain of $g(x)$: $(-\infty, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+1}{x^2 - 1}$

denominator $\neq 0$

$x^2 - 1 \neq 0$

$(x-1)(x+1) \neq 0$

So, domain of $\left(\frac{f}{g}\right)(x)$

$\{x \mid x = \mathbb{R}, x \neq -1, 1\}$

6. Let $f(x) = \sqrt[3]{x+4} - 5$ Find the inverse function of $f(x)$.

$y = \sqrt[3]{x+4} - 5$

$x = \sqrt[3]{y+4} - 5$

$x + 5 = \sqrt[3]{y+4}$

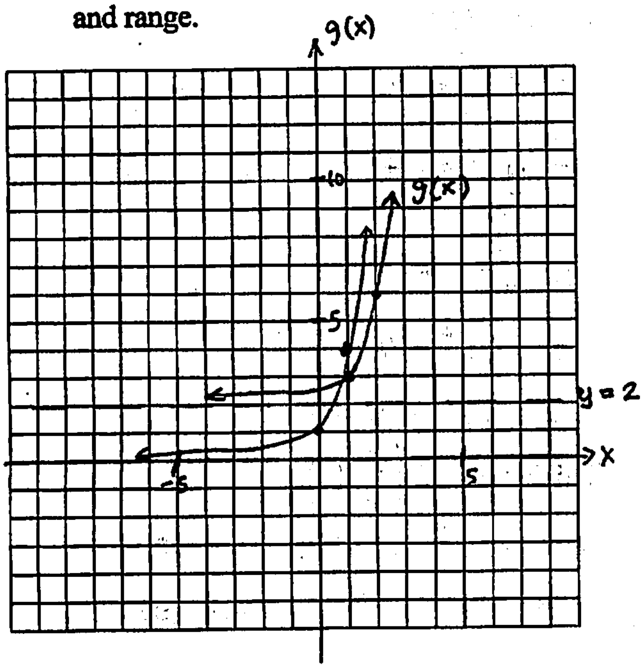
$(x+5)^3 = (\sqrt[3]{y+4})^3$

$(x+5)^3 = y + 4$

$(x+5)^3 - 4 = y$

$f^{-1}(x) = (x+5)^3 - 4$ //

7. Let $g(x)=4^{x-1}+2$. (i) Graph the function by transformation, (ii) Label asymptote, (iii) State domain and range.



$y = 4^{x}$

$y = 4^{x-1}$ shift right 1 unit

$y = 4^{x-1} + 2$ shift up 2 units

Domain : $(-\infty, \infty)$

Range : $(2, \infty)$ //

8. Solve: $\log_2(x-7)+\log_2 x=3$.

$\log_2 [(x-7)\cdot x] = 3$

$2^3 = x(x-7)$

$8 = x^2 - 7x$

$0 = x^2 - 7x - 8$

$0 = (x-8)(x+1)$

$x - 8 = 0$ $\quad x+1=0$

$x = 8$ $\quad \cancel{x = -1}$

$\{8\}$ //

9. Solve: $\log(x-6)-\log(x-2)=\log\left(\frac{5}{x}\right)$.

$$\log\left(\frac{x-6}{x-2}\right) = \log\left(\frac{5}{x}\right)$$

$$\frac{x-6}{x-2} = \frac{5}{x}$$

$$(x-6)\cdot x = 5(x-2)$$

$$x^2-6x = 5x-10$$

$$x^2-11x+10 = 0$$

$$(x-10)(x-1) = 0$$

$$x-10=0 \qquad x-1=0 \qquad \{10\}$$

$$x=10 \qquad \cancel{x=1}$$

10. If not checked, the population of a colony of bed bugs will grow exponentially at a rate of 65% per week. If a colony currently has 50 bedbugs, how many will there be in 6 weeks?

$k = 65\% = 0.65$

$A_0 = 50$

$t = 6$

$A = ?$

$$A = A_0 e^{kt}$$

$$= 50 \cdot e^{(0.65)(6)}$$

$$\approx 2471$$