

Show all necessary steps clearly, neatly, systematically to receive full credit.

1. (3 pts) Subtract  $-c^2d^2 + 5c^2d - 10cd^2$  from  $6c^2d^2 + 4c^2d - 5cd^2$ .

$$= (6c^2d^2 + 4c^2d - 5cd^2) - (-c^2d^2 + 5c^2d - 10cd^2)$$

$$= \underline{6c^2d^2 + 4c^2d - 5cd^2} + \underline{c^2d^2 - 5c^2d + 10cd^2}$$

$$= 7c^2d^2 - c^2d + 5cd^2 //$$

2. (3 pts) Multiply:  $(3x+2)(x^2-3x+6)$ .

$$= 3x^3 - 9x^2 + 18x + 2x^2 - 6x + 12$$

$$= 3x^3 - 7x^2 + 12x + 12 //$$

3. (3 pts) Factor completely:  $1 - x - 3z + 3xz$ .

$$= 1(1-x) - 3z(1-x)$$

$$= (1-x)(1-3z) //$$

4. (3 pts) Factor completely:  $x^4 - 16$ .

$$= (x^2 - 4)(x^2 + 4)$$

$$= (x-2)(x+2)(x^2+4) //$$

5. (3 pts) Factor completely:  $4d^7 + 4d^4$ .

$$= 4d^4(d^3 + 1)$$

$$= 4d^4(d+1)(d^2 - d + 1) //$$

6. (3 pts) Factor completely:  $27r^2st + 90rst - 72st$ .

$$= 9st(3r^2 + 10r - 8)$$

$$= 9st(3r - 2)(r + 4) //$$

7. (5 pts) Divide:  $(-2a + 3a^3 - 8) \div (a^2 + 5)$ .

$$a^2 + 5 \overline{) 3a^3 + 0a^2 - 2a - 8}$$

$$\underline{-(3a^3 + 15a)}$$

$$-17a - 8$$

$$3a + \frac{-17a - 8}{a^2 + 5} //$$

8. (5 pts) Perform indicated operation:

$$\frac{2a^2 - 5a - 3}{4a^3 - 36a} \div \frac{2a^2 + 5a + 2}{2a^2 + 5a - 3}$$

$$= \frac{2a^2 - 5a - 3}{4a^3 - 36a} \cdot \frac{2a^2 + 5a - 3}{2a^2 + 5a + 2}$$

$$= \frac{\overset{1}{(2a+1)}\overset{1}{(a-3)}}{4a\overset{1}{(a-3)}\overset{1}{(a+3)}} \cdot \frac{\overset{1}{(2a-1)}\overset{1}{(a+3)}}{\overset{1}{(2a+1)}\overset{1}{(a+2)}}$$

$$= \frac{2a-1}{4a(a+2)} //$$

9. (5 pts) Perform indicated operation:

$$\frac{6}{a^2 - 9} - \frac{5}{a^2 - a - 6}$$

$$= \frac{6}{(a-3)(a+3)} - \frac{5}{(a-3)(a+2)}$$

$$= \frac{6 \cdot (a+2) - 5(a+3)}{(a-3)(a+3)(a+2)}$$

$$= \frac{6a + 12 - 5a - 15}{(a-3)(a+3)(a+2)}$$

$$= \frac{a - 3}{(a-3)(a+3)(a+2)}$$

$$= \frac{1}{(a+3)(a+2)} //$$

side

$$4a^3 - 36a$$

$$= 4a(a^2 - 9)$$

$$= 4a(a-3)(a+3)$$

10. (5 pts) Solve:  $12x^2 = 5 - 4x$ .

$$12x^2 + 4x - 5 = 0$$

$$(2x - 1)(6x + 5) = 0$$

$$\begin{array}{l|l} 2x - 1 = 0 & 6x + 5 = 0 \\ 2x = 1 & 6x = -5 \\ x = \frac{1}{2} & x = -\frac{5}{6} \end{array}$$

$$\left\{ -\frac{5}{6}, \frac{1}{2} \right\} //$$

11. (5 pts) Simplify:  $\frac{\frac{8}{r+3}}{\frac{4}{r-2} - \frac{2}{r^2+r-6}}$ .

$$= \frac{\frac{8}{r+3}}{\frac{4}{r-2} - \frac{2}{(r+3)(r-2)}}$$

$$= \frac{\frac{8}{r+3}}{\frac{4}{r-2} - \frac{2}{(r+3)(r-2)}} \cdot \frac{(r+3)(r-2)}{(r+3)(r-2)}$$

$$= \frac{8(r-2)}{4(r+3) - 2}$$

$$= \frac{8(r-2)}{4r + 12 - 2}$$

$$= \frac{8(r-2)}{4r + 10}$$

$$= \frac{8(r-2)}{2(2r+5)}$$

$$= \frac{4(r-2)}{2r+5} //$$

12. (5 pts) Perform indicated operation:  $\frac{3}{a^2 - 2a} - \frac{1}{4 - 2a}$ .

$$= \frac{3}{a(a-2)} - \frac{1}{-2(a-2)}$$

$$= \frac{3}{a(a-2)} + \frac{1}{2(a-2)}$$

$$= \frac{3 \cdot 2 + 1 \cdot a}{2a(a-2)}$$

$$= \frac{6 + a}{2a(a-2)} //$$

13. (5 pts) Solve for  $b$ :  $H = \frac{2ab}{a+b}$ .

$$(a+b) \cdot H = \left( \frac{2ab}{a+b} \right) \cdot (a+b)$$

$$aH + bH = 2ab$$

$$aH = 2ab - bH$$

$$aH = b(2a - H)$$

$$\frac{aH}{2a - H} = b //$$

14. (6 pts) Simplify:  $\frac{(-2a^4bc^{-6})^{-3}}{(a^{-3}b^2c^{-2})^4}$ .

$$= \frac{(-2)^{-3} (a^4)^{-3} (b)^{-3} (c^{-6})^{-3}}{(a^{-3})^4 (b^2)^4 (c^{-2})^4}$$

$$= \frac{(-2)^{-3} a^{-12} b^{-3} c^{18}}{a^{-12} b^8 c^{-8}}$$

$$= \frac{a^{-12-(-12)} b^{-3-8} c^{18-(-8)}}{(-2)^3}$$

$$= \frac{a^0 b^{-11} c^{26}}{-8}$$

$$= \frac{c^{26}}{-8 b^{11}} //$$

15. (7 pts) Solve:  $s(2s+7) = (s+1)^2 + 71 - s$ .

$$2s^2 + 7s = s^2 + 2s + 1 + 71 - s$$

$$2s^2 + 7s = s^2 + s + 72$$

$$s^2 + 6s - 72 = 0$$

$$(s + 12)(s - 6) = 0$$

$$s + 12 = 0 \quad | \quad s - 6 = 0$$

$$s = -12 \quad | \quad s = 6$$

$$\{-12, 6\} //$$

16. (7 pts) Solve:  $\frac{x+3}{x-5} + \frac{2x^2+6}{x^2-7x+10} = \frac{3x}{x-2}$ .

multiply by LCD to both sides of equation to clear fractions.

$$\frac{x+3}{x-5} + \frac{2x^2+6}{(x-5)(x-2)} = \frac{3x}{x-2}$$

$$\text{LCD} = (x-5)(x-2); x \neq 5, 2$$

$$\rightarrow (x+3)(x-2) + 2x^2+6 = 3x(x-5)$$

$$x^2 + x - 6 + 2x^2 + 6 = 3x^2 - 15x$$

$$3x^2 + x = 3x^2 - 15x$$

$$x = -15x$$

$$16x = 0$$

$$x = 0$$

{0} //

17. (8 pts) A faucet can fill a garage sink in 2 minutes. It takes 3 minutes for the drain to empty the sink when it is full. How long will it take to fill the sink if the drain is open and the faucet is on?

①

	r	k	t	≠	job done
fill	$\frac{1}{2}$		2		1
drain		$\frac{1}{3}$	3		1

x = time take to fill the sink when drain is open and the faucet is on.

②  $\left(\frac{1}{2} - \frac{1}{3}\right) \cdot x = 1$

$$\frac{x}{2} - \frac{x}{3} = 1$$

$$6 \cdot \left(\frac{x}{2} - \frac{x}{3}\right) = (1) \cdot 6$$

$$3x - 2x = 6$$

$$x = 6$$

③ it will take 6 minutes //

18. (8 pts) A small plane pulling a banner can fly at a rate of 75 mph in calm air. Flying down the coast, with a tailwind, the plane flew 40 miles in the same time that it took to fly 35 miles up the coast, into a headwind. Find the rate of the wind.

①

	r	t	d
tailwind	$75+x$	$\frac{40}{75+x}$	40
headwind	$75-x$	$\frac{35}{75-x}$	35

rate of wind =  $x$

side

$$\begin{array}{r} \cancel{y} \cancel{2} 75 \\ \times 35 \\ \hline 1375 \\ + 2250 \\ \hline 2625 \end{array}$$

②  $\frac{40}{75+x} = \frac{35}{75-x}$

$$\begin{array}{r} 5 \\ 75 \overline{) 375} \\ \underline{-375} \\ 0 \end{array}$$

$$40(75-x) = 35(75+x)$$

$$3000 - 40x = 2625 + 35x$$

$$3000 = 2625 + 75x$$

$$375 = 75x$$

$$5 = x$$

③ rate of wind is 5 mph.

19. (8 pts) The wind force on a vertical surface varies jointly as the area of the surface and the square of the wind's velocity. If a 10 mph wind exerts a force of 1.98 pounds on the sign with  $4.5 \text{ ft}^2$  area, find the force on the sign if the wind is blowing at 80 mph.

① force =  $F$   
 area =  $A$   
 velocity =  $V$

②  $F = kAV^2$

$F = 1.98$ $A = 4.5$ $V = 10$ $k = ?$	$F = kAV^2$ $1.98 = k \cdot (4.5)(10)^2$ $1.98 = 450k$ $\frac{1.98}{450} = k$ $\frac{198}{45000} = k$	$F = ?$ $A = 4.5$ $V = 80$ $k = \frac{198}{45000}$	$F = kAV^2$ $F = \frac{198}{45000} \cdot (4.5) \cdot (80)^2$ $F = \frac{198}{450} \cdot (4.5) \cdot \frac{64}{1}$ $F = \frac{198}{100} \cdot \frac{4.5}{1} \cdot \frac{64}{1}$ $F = 126.72$
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③ the force on the sign is 126.72 lb.