Math 165 Test 3 Name: Key

1. A Pizza shop owner determines the number of pizzas that are delivered each day. Find the mean, variance and standard deviation for the distribution shown.

Number of deliveries X	35	36	37	38	39
Probability $P(X)$	0.1	0.2	0.3	0.3	0.1
$X \cdot P(x)$	3.5	7.2	11.1	11.4	3.9
X^2	1225	1296	1369	1444	1521
$X^2 \cdot P(X)$	122.5	259.2	410.7	433.2	152.1

$$\mu = \Sigma[X \cdot P(X)] = 37.1$$

$$\sigma^2 = \Sigma[x^2 \cdot P(x)] - \mu^2 = 1377.7 - 1376.41 = 1.29$$

$$\sigma = \sqrt{\Sigma[x^2 \cdot P(x)] - \mu^2} = \sqrt{1.29} \approx 1.1$$

2. A lottery offers one \$1000 prize, two \$500 prizes, three \$250 prizes, four \$125 prizes, and five \$100 prizes. One thousand tickets are sold at \$3 each. Find the expectation if a person buys one ticket.

		x	$P(x)$	$x \cdot P(x)$
win	\$1000 prize	997	$\frac{1}{1000}$	$\frac{997}{1000}$
	\$500 prize	497	$\frac{2}{1000}$	$\frac{994}{1000}$
	\$250 prize	247	$\frac{3}{1000}$	$\frac{741}{1000}$
	\$125 prize	122	$\frac{4}{1000}$	$\frac{488}{1000}$
	\$100 prize	97	$\frac{5}{1000}$	$\frac{485}{1000}$
lose	\$3 ticket	-3	$\frac{985}{1000}$	$-\frac{2955}{1000}$

$$E(x) = \Sigma[x \cdot P(x)]$$

$$= \frac{997 + 994 + 741 + 488 + 485 - 2955}{1000}$$

$$= 0.75$$

3. Approximately 10.3% of American high school students drop out of school before graduation. Choose 10 students entering high school at random. Find the probability that

a. Exactly three drop out

$p = 0.103$

$q = 0.897$

$n = 10$

$$P(x=3) = {}_{10}C_3 \, (0.103)^3 \, (0.897)^7$$

$$\approx 0.0613$$

b. No More than two drop out

$$P(X \leq 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= {}_{10}C_0 \, (0.103)^0 \, (0.897)^{10} + {}_{10}C_1 \, (0.103)^1 (0.897)^9 + {}_{10}C_2 \, (0.103)^2 (0.897)^8$$

$$\approx 0.9245$$

4. Thirty-two percent of adult Internet users have purchased products or services online. For a random sample of 200 adult Internet users, find the mean, variance, and standard deviation for the number who have purchased goods or services online.

$p = 0.32$

$q = 0.68$

$n = 200$

$$\mu = n \cdot p = (200)(0.32) = 64$$

$$\sigma = n \cdot p \cdot q = (200)(0.32)(0.68) = 43.52$$

$$\sigma^2 = \sqrt{npq} = \sqrt{(200)(0.32)(0.68)} = 6.6$$

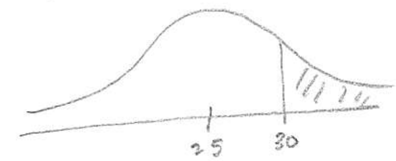
5. The average commute to work (one way) is 25 minutes according to the 2005 American Community Survey. If we assume that commuting times are normally distributed and the standard deviation is 6.1 minutes, what is the probability that a randomly selected commuter spends more than 30 minutes commuting one way?

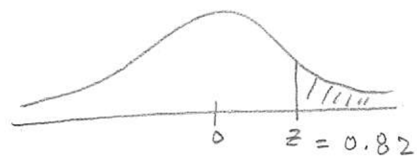
$\mu = 25$

$\sigma = 6.1$

$$P(X > 30)$$
$$= P(z > 0.82)$$
$$= 1 - 0.7939$$
$$= 0.2061$$

$$z = \frac{X - \mu}{\sigma}$$
$$= \frac{30 - 25}{6.1}$$
$$\approx 0.82$$

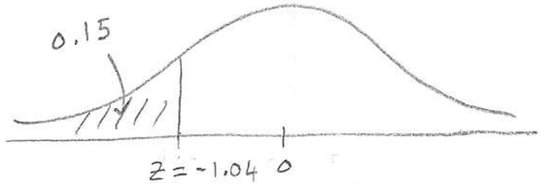




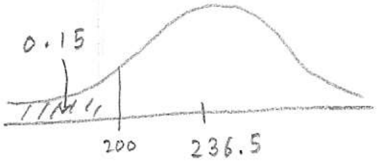
6. The average retail price of gasoline (all types) for the first half of 2009 was 236.5 cents. What would the standard deviation have to be in order for a 15% probability that a gallon of gas costs less than $2.00?

$\mu = 236.5$

$X = 200$



$P(X < 200) = 0.15$



$$z = \frac{X - \mu}{\sigma}$$

$$-1.04 = \frac{200 - 236.5}{\sigma}$$

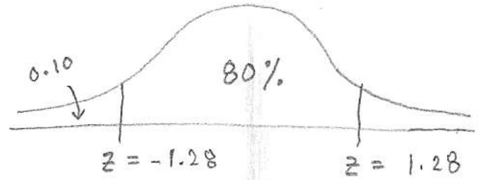
$$\sigma = \frac{-36.5}{-1.04}$$

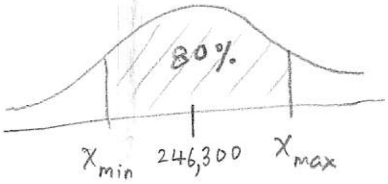
$$\sigma \approx 35.1$$

7. If the average price of a new one-family home is $246,300 with a standard deviation of $15,000, find the minimum and maximum prices of the houses that a contractor will build to satisfy the middle 80% of the market. Assume that the variable is normally distributed.

$\mu = 246,300$

$\sigma = 15,000$





$$z = \frac{X_{min} - \mu}{\sigma} \qquad z = \frac{X_{max} - \mu}{\sigma}$$

$$-1.28 = \frac{X_{min} - 246300}{15000} \qquad 1.28 = \frac{X_{max} - 246300}{15000}$$

$$-19200 = X_{min} - 246300 \qquad 19200 = X_{max} - 246300$$

$$227100 = X_{min} \qquad 265500 = X_{max}$$

8. Americans drank an average of 23.3 gallons of bottled water per capita in 2008. If the standard deviation is 2.7 gallons and the variable is normally distributive, find the probability that a randomly selected American drank more than 25 gallons of bottled water.

$\mu = 23.3$

$\sigma = 2.7$

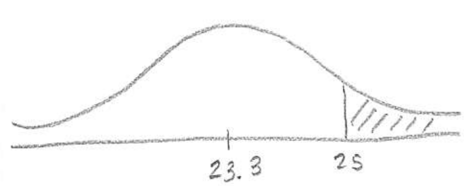
$P(x > 25)$

$= P(z > 0.63)$

$= 1 - 0.7357$

$= 0.2643$

$$z = \frac{x - \mu}{\sigma} = \frac{25 - 23.3}{2.7} \approx 0.63$$



9. A recent study of a life times of cell phones found the average is 24.3 months. The standard deviation is 2.6 months. If a company provides its 33 employees with a cell phone, find the probability that the mean lifetime of these phones will be less than 23.8 months. Assume cell phone life is a normally distributed variable.

$\mu = 24.3$

$\sigma = 2.6$

$n = 33$

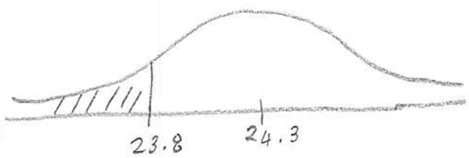
$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{2.6}{\sqrt{33}}$

$\mu_{\bar{x}} = \mu = 24.3$

$P(x < 23.8) = P(z < -1.10) = 0.1357$

$z = \dfrac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$

$z = \dfrac{23.8 - 24.3}{2.6/\sqrt{33}} \approx -1.10$



10. Assume that the mean systolic blood pressure of normal adults is 120 millimeters of mercury and the standard deviation is 5.6. Assume the variable is normally distributed.

a. If an individual is selected, find the probability that the individual's pressure will be between 120 and 121.8.

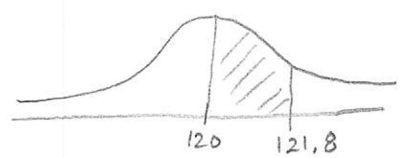
$\mu = 120$

$\sigma = 5.6$

$P(120 < x < 121.8) = P(0 < z < 0.32) = 0.6255 - 0.5 = 0.1255$

$z = \dfrac{x - \mu}{\sigma} = \dfrac{120 - 120}{5.6} = 0$

$z = \dfrac{x - \mu}{\sigma} = \dfrac{121.8 - 120}{5.6} \approx 0.32$



b. If a sample of 30 adults is randomly selected, find the probability that the sample mean will be between 120 and 121.8.

$\mu = 120$

$\sigma = 5.6$

$n = 30$

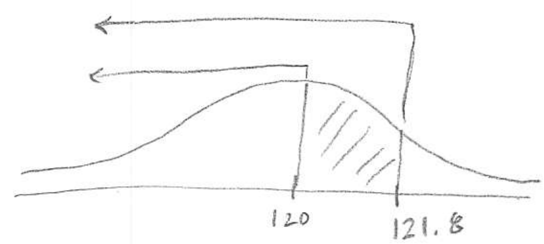
$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{5.6}{\sqrt{30}} \approx$

$\mu_{\bar{x}} = \mu = 120$

$P(120 < x < 121.8) = P(0 < z < 1.76) = 0.9608 - 0.5 = 0.4608$

$z = \dfrac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \dfrac{120 - 120}{5.6/\sqrt{30}} = 0$

$z = \dfrac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \dfrac{121.8 - 120}{5.6/\sqrt{30}} \approx 1.76$



11. According to recent surveys, 60% of households have personal computers. If a random sample of 180 households is selected, what is the probability that more than 60 have a personal computer?

$p = 0.6$

$q = 0.4$

$n = 180$

$\begin{cases} np = 108 \geq 5 \\ np = 72 \geq 5 \end{cases}$

can approximate as normal.

$\mu = np = 108$

$\sigma = \sqrt{npq} \approx 6.5$

$P(X > 60)$

$= P(X > 60.5)$

$= P(Z > -7.3)$

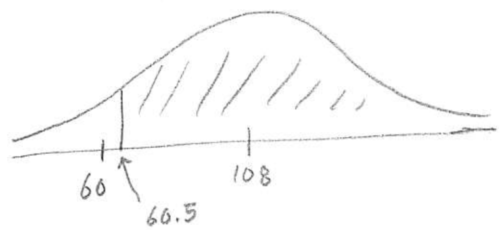
$= 1 - 0.0002$

$= 0.9998$

$z = \dfrac{X - \mu}{\sigma}$

$= \dfrac{60.5 - 108}{6.5}$

≈ -7.3



12. Seventy-eight percent of U.S. homes have a telephone answering device. In a random sample of 250 homes, what is the probability that at most 40 have a telephone answering device?

$p = 0.78$

$q = 0.22$

$n = 250$

$\begin{cases} np = 195 \geq 5 \\ nq = 55 \geq 5 \end{cases}$

can approximate as normal.

$\mu = np = 195$

$\sigma = \sqrt{npq} \approx 6.5$

$P(X \leq 40)$

$= P(X < 40.5)$

$= P(Z < -23.77)$

$= 0.0002$

$z = \dfrac{X - \mu}{\sigma}$

$= \dfrac{40.5 - 195}{6.5}$

≈ -23.77