Math 165 Test 4 Name: ____________________

1. Eight chemical elements do not have isotopes (different forms of the same element having the same atomic number but different atomic weights). A random sample of 30 of the elements that do have isotopes showed a mean number of 19.63 isotopes per element and the population a standard deviation of 18.73. Estimate the true mean number of isotopes for all elements with isotopes with 90% confidence.

$n = 30$

$\bar{x} = 19.63$

$\sigma = 18.73$

$C.L = 0.90$

$\alpha = 0.10$

$\frac{\alpha}{2} = 0.05$

$z_{\frac{\alpha}{2}} = 1.645$

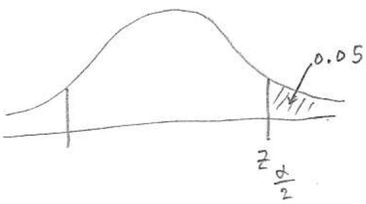
$E = z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right)$

$= 1.645\left(\frac{18.73}{\sqrt{30}}\right)$

≈ 5.625265854

$\bar{x} - E < \mu < \bar{x} + E$

$14.005 < \mu < 25.255$ //



2. A study found that 73% of prekindergarten children ages 3 to 5 whose mothers had a bachelor's degree or higher were enrolled in center-based early childhood care and education programs. How large a sample is needed to estimate the true proportion within 3 percentage points with 95% confidence?

$\hat{p} = 0.73$

$\hat{q} = 0.27$

$E = 0.03$

$C.L = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

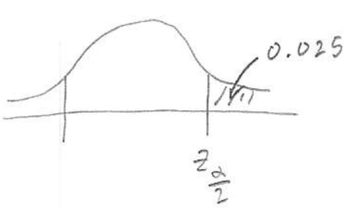
$z_{\frac{\alpha}{2}} = 1.96$

$n = \hat{p}\hat{q}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2$

$= (0.73)(0.27)\left(\frac{1.96}{0.03}\right)^2$

≈ 841.3104

≈ 842 //



3. The lengths (in min.) of a random selection of popular children's animated films are listed below. Estimate the true mean length of all children's animated films with 95% confidence. Assume that population is normally distributed.

93	83	76	92	77	81	78	100	78	76	75

σ unknown

$n = 11$

$\bar{x} = 82.6$

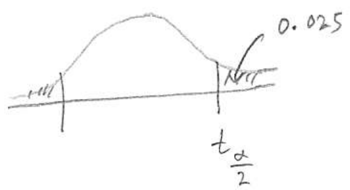
$s = 8.5$

$C.L = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$d.f = n - 1$
$= 10$



$t_{\frac{\alpha}{2}} = 2.228$

$E = t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$

$= 2.228\left(\frac{8.5}{\sqrt{11}}\right)$

$\approx 5.71002184 4$

$\bar{x} - E < \mu < \bar{x} + E$

$76.9 < \mu < 88.3$ //

4. In a study of 200 accidents that required treatment in an emergency room, 80 occurred at work. Find 99% confidence interval of the true proportion of accidents that occurred at work.

$n = 200$

$x = 80$

$\hat{p} = \frac{x}{n}$

$= \frac{80}{200}$

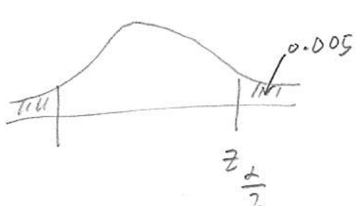
$= 0.4$

$\hat{q} = 0.6$

$C.L = 0.99$

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$



$z_{\frac{\alpha}{2}} = 2.575$

$E = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 2.575\left(\sqrt{\frac{(0.4)(0.6)}{200}}\right)$

≈ 0.0892006166

$\hat{p} - E < p < \hat{p} + E$

$0.3 < p < 0.5$ //

5. A researcher wishes to estimate within $25 the average cost of postage a community college spends in one year. If she wishes to be 95% confident, how large of a sample would be necessary if the population standard deviation is $80.

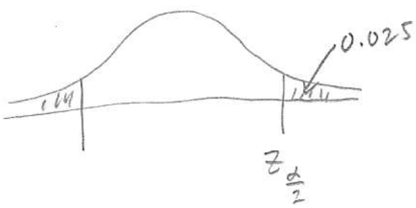
$E = 25$

$\sigma = 80$

$C.L = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$



$z_{\frac{\alpha}{2}} = 1.96$

$$n = \left(\frac{z_{\frac{\alpha}{2}} \cdot \sigma}{E}\right)^2$$

$$= \left(\frac{1.96 \times 80}{25}\right)^2$$

$$\approx 39.337984$$

$$\approx 40$$

6. A sample of 20 automobiles has a pollution by-product release standard deviation of 2.3 ounces when one gallon of gasoline is used. Find the 90% confidence interval of the population standard deviation.

$n = 20$

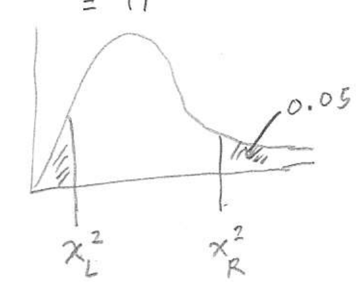
$s = 2.3$

$C.L = 0.90$

$\alpha = 0.10$

$\frac{\alpha}{2} = 0.05$

$d.f = n - 1$
$= 19$



$\chi^2_L = 10.117$

$\chi^2_R = 30.144$

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{\frac{(19)(2.3)^2}{30.144}} < \sigma < \sqrt{\frac{(19)(2.3)^2}{10.117}}$$

$$1.83 < \sigma < 3.15$$

7. Based on information from the U.S. Census Bureau, the mean travel time to work in minutes for all workers 16 years old and older was 25.3 minutes. A large company with offices in several states randomly sampled 100 of its workers to ascertain their commuting times. The sample mean was 23.9 minutes, and the population standard deviation is 6.39 minutes. At the 0.01 level of significance can it be concluded that the mean commuting time is less for this particular company?

① mean commuting time is less for this particular company.

② $H_0 : \mu = 25.3$

$H_1 : \mu < 25.3$ (claim)

③ Test-Value :

$n = 100$

$\bar{x} = 23.9$

$\sigma = 6.39$

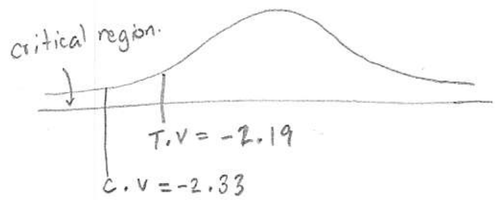
$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{23.9 - 25.3}{6.39/\sqrt{100}} \approx -2.19$$

④ Critical Value :

$\alpha = 0.01$

C.V $= -2.33$

⑤ Informal Conclusion :



do not reject H_0.

⑥ Conclusion :

we don't have enough evidence to support that mean commuting time is less for this company.

8. Nationwide 13.7% of employed wage and salary workers are union members (down from 20.1% in 1983). A random sample of 300 local wage and salary workers showed that 50 belonged to a union. At $\alpha = 0.01$, is there sufficient evidence to conclude that the proportion of union membership differs from 13.7%?

① the proportion of union membership differs from 13.7%.

② $H_0 : p = 0.137$

$H_1 : p \neq 0.137$ (claim)

③ Test-Value :

$n = 300$

$x = 50$

$\hat{p} = \frac{x}{n} = \frac{50}{300} = 0.167$

$\hat{q} = 0.833$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.167 - 0.137}{\sqrt{\frac{(0.137)(0.863)}{300}}} \approx 1.51$$

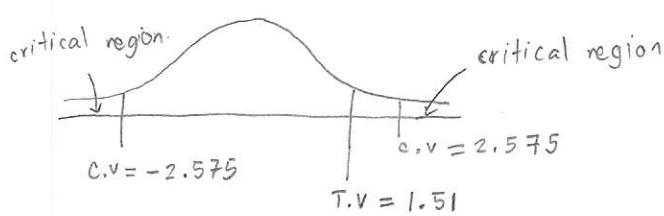
④ Critical Value :

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

C.V $= 2.575$

⑤ Informal Conclusion :



do not reject H_0

⑥ Conclusion ;

we do not have sufficient evidence to support that the proportion of union membership differs from 13.7%.

9. Nationwide, the average salary of actuaries who achieve the rank of Fellow is \$150000. An insurance executive wants to see how this compares with Fellows within his company. He checks the salaries of eight Fellows and finds the average salary to be \$155500 with a standard deviation of \$15000. Can he conclude that Fellows in his company make more than the nation average, using 5% level of significance?

① Fellows in his company make more than the nation average.

② $H_0: \mu = 150000$
$H_1: \mu > 150000$ (claim)

③ Test Value:

$n = 8$

$\bar{x} = 155500$

$s = 15000$

σ is unknown

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$= \frac{155500 - 150000}{\frac{15000}{\sqrt{8}}}$$

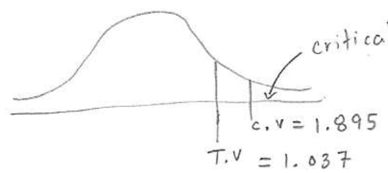
$$\approx 1.037$$

④ critical value:

$\alpha = 0.05$

$d.f = n - 1$
$= 7$

$c.v = 1.895$

⑤ 

do not reject H_0.

⑥ we do not have enough evidence to support that the fellows in his company make more than nation average.

10. A random sample of the number of games played by individual NBA scoring leaders is found below. Is there sufficient evidence to conclude that he variance in games played differs from 40? Use $\alpha = 0.05$.

72	79	80	74	82	79	82	78	60	75

① variance in games played differs from 40.

② $H_0: \sigma^2 = 40$
$H_1: \sigma^2 \neq 40$ (claim)

③ Test Value:

$n = 10$

$s = 6.6$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$= \frac{(10-1)(6.6)^2}{40}$$

$$\approx 9.801$$

④ Critical Value:

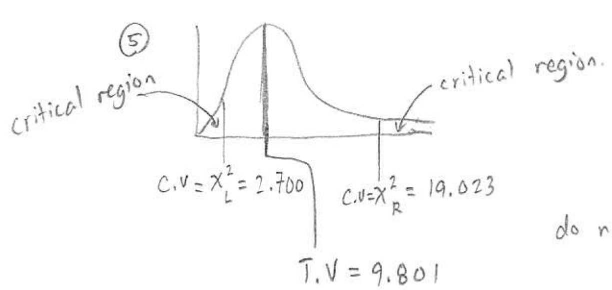
$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$d.f = n - 1$
$= 9$

$\chi^2_L = 2.700$

$\chi^2_R = 19.023$

⑤ 

do not reject H_0

⑥ We do not have enough evidence to support that variance in games played differs from 40.

Extra Credit: For hypothesis testing problems, find *P-Value*, informal conclusion, and conclusion.

7	P-Value = 0.0143 α = 0.01	P-value > α do not reject H_0	we do not have enough evidence to support that mean commuting time is less for this company.
8	P-Value = 2(0.0655) α = 0.01	p-value > α do not reject H_0	we do not have enough evidence to support that the proportion of union membership differs from 13.7%.
9	P-Value > 0.10 α = 0.05	P-value > α do not reject H_0	we do not have enough evidence to support that Fellows in his company make more than nation average.
10	P-value > 2(0.10) α = 0.05	P-value > α do not reject H_0	we do not have enough evidence to support that he variance in the games played differs from 40.