

Show all necessary work neatly, clearly, and systematically. Any understatement and/or incorrect statement may be penalized.

1. (1) Factorize completely:  $x^2 - 5xy - 14y^2$

$$= (x + 2y)(x - 7y)$$

2. (1) Factorize completely:  $64x^2 - 48x + 9$

$$= (8x - 3)^2$$

3. (2) Factorize completely:  $3x^3 + 3x^2y - 15xy^2$

$$= 3x(x^2 + xy - 5y^2)$$

4. (2) Factorize completely:  $45a^3b - 78a^2b^2 + 24ab^3$

$$= 3ab(15a^2 - 26ab + 8b^2)$$

$$= 3ab(3a - 4b)(5a - 2b)$$

5. (2) Factorize completely:  $15a^2 - 14ab - 8b^2$

$$= (3a - 4b)(5a + 2b)$$

6. (3) Factorize completely:  $2x^3 + 3x^2 - 8x - 12$

$$= x^2(2x + 3) - 4(2x + 3)$$

$$= (2x + 3)(x^2 - 4)$$

$$= (2x + 3)(x - 2)(x + 2)$$

7. (3) Factorize completely:  $x^4 - 81$

$$= (x^2)^2 - (9)^2$$

$$= (x^2 - 9)(x^2 + 9)$$

$$= (x - 3)(x + 3)(x^2 + 9)$$

8. (3) Solve:  $2x^2 = 6 + 3x$

$$2x^2 - 3x - 6 = 0$$

9. (3) Solve for r:  $A = P(1 + rt)$

$$A = P + Prt$$

$$A - P = Prt$$

$$\frac{A - P}{Pt} = r$$

10. (3) Simplify:  $\frac{5 + 4x - x^2}{3x^2 - 30x + 75}$

$$= \frac{-1(5 - x)(1 + x)}{3(x - 5)(x - 5)}$$

$$= \frac{-(1 + x)}{3(x - 5)}$$

side

$$3x^2 - 30x + 75$$

$$3(x^2 - 10x + 25)$$

$$3(x - 5)(x - 5)$$

11. (4) Simplify:  $\frac{6}{3x+1} - \frac{4}{2x-7}$

$$= \frac{6(2x-7) - 4(3x+1)}{(3x+1)(2x-7)}$$

$$= \frac{12x - 42 - 12x - 4}{(3x+1)(2x-7)}$$

$$= \frac{-46}{(3x+1)(2x-7)}$$

12. (4) Simplify:  $\frac{1 - \frac{7}{x} + \frac{12}{x^2}}{1 + \frac{1}{x} - \frac{20}{x^2}} \cdot \frac{x^2}{x^2}$

$$= \frac{x^2 - 7x + 12}{x^2 + x - 20}$$

$$= \frac{(x-3)(x-4)}{(x-4)(x+5)}$$

$$= \frac{x-3}{x+5}$$

14. (5) Solve:  $5x = 11 - \frac{2}{x}$  LCD =  $x$ ;  $x \neq 0$

$$x(5x) = \left(11 - \frac{2}{x}\right) \cdot x$$

$$5x^2 = 11x - 2$$

$$5x^2 - 11x + 2 = 0$$

$$(x-2)(5x-1) = 0$$

$$\begin{array}{l|l} x-2=0 & 5x-1=0 \\ x=2 & 5x=1 \\ & x=\frac{1}{5} \end{array}$$

$$\left\{ \frac{1}{5}, 2 \right\}$$

13. (4) Solve:  $x(x+3) = 28$

$$x^2 + 3x = 28$$

$$x^2 + 3x - 28 = 0$$

$$(x-4)(x+7) = 0$$

$$\begin{array}{l|l} x-4=0 & x+7=0 \\ x=4 & x=-7 \end{array}$$

$$\{-7, 4\}$$

15. (5) Simplify:  $\frac{\frac{4}{2x+1} - \frac{2}{x}}{\frac{6}{x} - \frac{3}{2x+1}} \cdot \frac{x(2x+1)}{x(2x+1)}$

$$= \frac{4 \cdot x - 2 \cdot (2x+1)}{6 \cdot (2x+1) - 3 \cdot x}$$

$$= \frac{4x - 4x - 2}{12x + 6 - 3x}$$

$$= \frac{-2}{9x + 6}$$

$$\begin{aligned}
 16. (5) \text{ Simplify: } & \frac{x-7}{x^2+2x-8} + \frac{5}{x+4} \\
 = & \frac{x-7}{(x-2)(x+4)} + \frac{5}{x+4} \\
 = & \frac{(x-7) + 5(x-2)}{(x-2)(x+4)} \\
 = & \frac{x-7+5x-10}{(x-2)(x+4)} \\
 = & \frac{6x-17}{(x-2)(x+4)}
 \end{aligned}$$

$$\begin{aligned}
 18. (6) \text{ Simplify: } & \frac{2y^2+5y-3}{y^2-2y-15} \div \frac{2y^2-5y+2}{y^2-25} \\
 = & \frac{2y^2+5y-3}{y^2-2y-15} \cdot \frac{y^2-25}{2y^2-5y+2} \\
 = & \frac{(y+3)(2y-1)}{(y+3)(y-5)} \cdot \frac{(y-5)(y+5)}{(y-2)(2y-1)} \\
 = & \frac{\cancel{(y+3)}\cancel{(2y-1)}}{\cancel{(y+3)}(y-5)} \cdot \frac{\cancel{(y-5)}(y+5)}{(y-2)\cancel{(2y-1)}} \\
 = & \frac{y+5}{y-2}
 \end{aligned}$$

$$\begin{aligned}
 17. (5) \text{ Simplify: } & \frac{x^2+2x-15}{x^2+12x+35} \cdot \frac{x^2+16x+63}{x^2-x-6} \\
 = & \frac{(x-3)(x+5)}{(x+5)(x+7)} \cdot \frac{(x+7)(x+9)}{(x+2)(x-3)} \\
 = & \frac{\cancel{(x-3)}\cancel{(x+5)}}{\cancel{(x+5)}(x+7)} \cdot \frac{\cancel{(x+7)}(x+9)}{(x+2)\cancel{(x-3)}} \\
 = & \frac{x+9}{x+2} //
 \end{aligned}$$

$$\begin{aligned}
 19. (6) \text{ Simplify: } & \frac{2x^2+5x+2}{2x^2+3x-2} \div \frac{3x^2+13x+4}{2x^2+7x-4} \\
 = & \frac{2x^2+5x+2}{2x^2+3x-2} \cdot \frac{2x^2+7x-4}{3x^2+13x+4} \\
 = & \frac{(x+2)(2x+1)}{(x+2)(2x-1)} \cdot \frac{(x+4)(2x-1)}{(x+4)(3x+1)} \\
 = & \frac{\cancel{(x+2)}(2x+1)}{\cancel{(x+2)}(2x-1)} \cdot \frac{\cancel{(x+4)}\cancel{(2x-1)}}{\cancel{(x+4)}(3x+1)} \\
 = & \frac{2x+1}{3x+1}
 \end{aligned}$$

20. (6) Solve:  $(2x-1)(x+5) = x^2 + 2x + 3$

$$2x^2 + 10x - x - 5 = x^2 + 2x + 3$$

$$2x^2 + 9x - 5 = x^2 + 2x + 3$$

$$x^2 + 7x - 8 = 0$$

$$(x-1)(x+8) = 0$$

$$\begin{array}{l|l} x-1=0 & x+8=0 \\ x=1 & x=-8 \end{array}$$

$$\{-8, 1\}$$

21. (7) Solve:  $\frac{1}{x+1} - \frac{x+1}{x^2-4} = \frac{x+2}{x^2-x-2}$

$$\frac{1}{x+1} - \frac{x+1}{(x-2)(x+2)} = \frac{x+2}{(x+1)(x-2)}$$

$$\text{LCD} = (x+1)(x-2)(x+2) ; x \neq -1, -2, 2$$

$$(x+1)(x+2)(x-2) \left( \frac{1}{x+1} - \frac{x+1}{(x-2)(x+2)} \right) = \left( \frac{x+2}{(x+1)(x-2)} \right) (x+1)(x+2)(x-2)$$

$$1 \cdot (x+2)(x-2) - (x+1)(x+1) = (x+2)(x+2)$$

$$(x^2 - 4) - (x^2 + 2x + 1) = x^2 + 4x + 4$$

$$x^2 - 4 - x^2 - 2x - 1 = x^2 + 4x + 4$$

$$-2x - 5 = x^2 + 4x + 4$$

$$0 = x^2 + 6x + 9$$

$$0 = (x+3)(x+3)$$

$$\begin{array}{l|l} x+3=0 & x+3=0 \\ x=-3 & \end{array}$$

$$\{-3\}$$

#21

	r	x	t	= job done
small	$\frac{1}{2x}$		2x	1
large	$\frac{1}{x}$		x	1

time together = 6

$$\left(\frac{1}{2x} + \frac{1}{x}\right) \cdot 6 = 1$$

$$\frac{6}{2x} + \frac{6}{x} = 1$$

$$2x \left(\frac{6}{2x} + \frac{6}{x}\right) = (1) \cdot 2x$$

$$6 + 12 = 2x$$

$$18 = 2x$$

$$9 = x$$

#22

	r	x	t	= d
BB	$x-2$		$\frac{12}{x-2}$	12
DD	x		$\frac{18}{x}$	18

$$\frac{12}{x-2} = \frac{18}{x}$$

$$x(x-2) \left(\frac{12}{x-2}\right) = \left(\frac{18}{x}\right) \cdot x(x-2)$$

$$12x = 18(x-2)$$

$$12x = 18x - 36$$

$$-6x = -36$$

$$x = 6$$