① $\ln\left[\dfrac{(x+1)^{\frac{1}{2}}}{x(x+1)^2}\right]$

$= \ln (x+1)^{\frac{1}{2}} - \ln\left[x(x+1)^2\right]$

$= \ln (x+1)^{\frac{1}{2}} - \left[\ln x + \ln (x+1)^2\right]$

$= \frac{1}{2}\ln(x+1) - \ln x - 2\ln(x+1)$

$= -\frac{3}{2}\ln(x+1) - \ln x$

---

② vertex

$x = -\frac{b}{2a} = -\frac{-3}{2(-1)} = -\frac{3}{2}$

$y = f(-\frac{3}{2}) = -\frac{11}{4}$

$$\begin{array}{r|rrr} -\frac{3}{2} & -1 & -3 & 4 \\ & & \frac{3}{2} & \frac{9}{4} \\ \hline & -1 & -\frac{3}{2} & \frac{25}{4} \end{array}$$

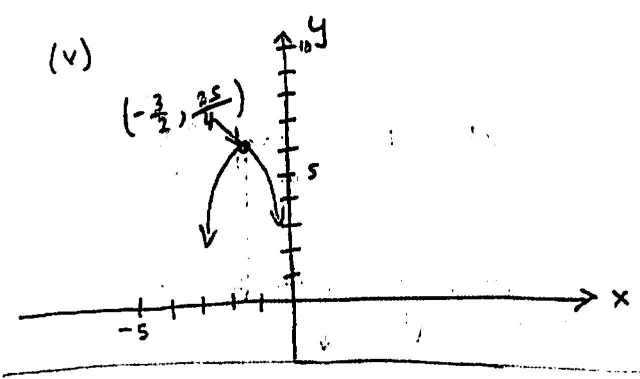
(i) $f(x) = -(x+\frac{3}{2})^2 + \frac{25}{4}$

(ii) $(-\frac{3}{2}, \frac{25}{4})$

(iii) $x = -\frac{3}{2}$

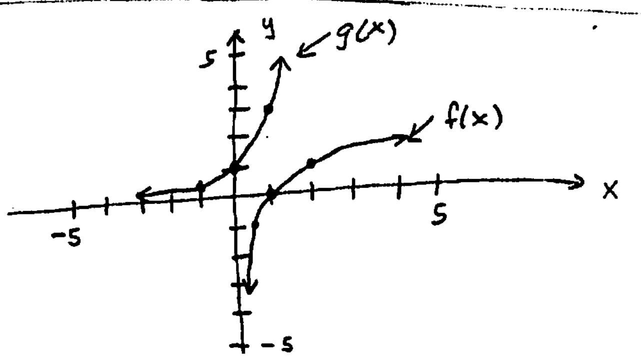
(iv) max function value $= \frac{25}{4}$

(v)



---

③



---

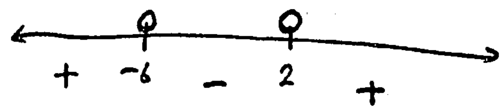
④ $x^2 + 4x - 12 > 0$

$(x+6)(x-2) > 0$

zeros

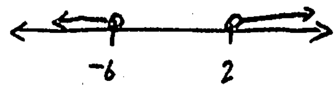
$x+6=0 \qquad x-2=0$

$x=-6 \qquad x=2$



(i) $(-\infty, -6) \cup (2, \infty)$

(ii)



---

⑤ $5^{2x-1} = 7$

$\log 5^{2x-1} = \log 7$

$(2x-1)\log 5 = \log 7$

$2x\log 5 - \log 5 = \log 7$

$2x\log 5 = \log 7 + \log 5$

$x = \dfrac{\log 7 + \log 5}{2\log 5}$

$x \approx 1.105$

---

⑥ $125^{x+1} = \frac{1}{25}$

$(5^3)^{x+1} = \frac{1}{5^2}$

$5^{3x+3} = 5^{-2}$

$3x+3 = -2$

$x = \frac{-5}{3}$

$x \approx -1.667$

⑦ $\log_5(x+1) - \log_5(x-1) = 2$

$\log_5\left(\frac{x+1}{x-1}\right) = 2$

$5^2 = \frac{x+1}{x-1}$

$25 = \frac{x+1}{x-1}$

$25(x-1) = x+1$

$25x - 25 = x+1$

$24x - 25 = 1$

$24x = 26$

$x = \frac{26}{24}$

$x = \frac{13}{12}$

---

⑧ $\log_8(x+3) + \log_8(x+5) = 1$

$\log_8(x+3)(x+5) = 1$

$8^1 = (x+3)(x+5)$

$8 = x^2+8x+15$

$0 = x^2+8x+7$

$0 = (x+7)(x+1)$

$x+7=0 \quad x+1=0$

$x \cancel{=} -7 \quad x=-1$

$\{-1\}$

⑨ $(f \circ g)(x) = f(g(x))$

$= f\left((x+2)^3-1\right)$

$= \sqrt[3]{(x+2)^3-1+1} - 2$

$= \sqrt[3]{(x+2)^3} - 2$

$= x+2-2$

$= x$

$(g \circ f)(x) = g(f(x))$

$= g\left(\sqrt[3]{x+1} - 2\right)$

$= \left(\sqrt[3]{x+1} - 2 + 2\right)^3 - 1$

$= \left(\sqrt[3]{x+1}\right)^3 - 1$

$= x+1-1$

$= x$

yes, f and g are invese of each other.

---

⑩ (a) $\left(\frac{f}{g}\right)(-2) = \frac{f(-2)}{g(-2)} = \frac{23}{-1} = -23$

side $f(-2) = 4(-2)^2 - 3(-2) + 1 = 23$

$g(-2) = 2(-2) + 3 = -1$

(b) $(f \circ g)(x) = f(g(x))$

$= f(2x+3)$

$= 4(2x+3)^2 - 3(2x+3) + 1$

$= 4(4x^2+12x+9) - 6x - 9 + 1$

$= 16x^2 + 48x + 36 - 6x - 9 + 1$

$= 16x^2 + 42x + 28$

⑪ $h(t) = vt - 16t^2$

$h(t) = 160t - 16t^2$

$h(t) = -16t^2 + 160t$

$\langle (\text{time}, \text{height}) \rangle$

vertex
$$\begin{cases} t = -\frac{b}{2a} = -\frac{160}{2(-16)} = 5 \\ h(5) = -16(5)^2 + 160(5) = 400 \end{cases}$$

⑫ $A = 2x$

$P = x$

$r = 0.04$

$n = 12$

$t = ?$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$2x = x \cdot \left(1 + \frac{0.04}{12}\right)^{12t}$$

$$2 = \left(1 + \frac{0.04}{12}\right)^{12t}$$

$$2 = (1.00\bar{3})^{12t}$$

$$\log 2 = \log (1.00\bar{3})^{12t}$$

$$\log 2 = 12t \log (1.00\bar{3})$$

$$\frac{\log 2}{12 \log (1.00\bar{3})} = t$$

$$17.35\ldots \approx t$$

17 yrs, 5 month $\approx t$