

Show all necessary work NEATLY, CLEARLY, and SYSTEMATICALLY. Any understatement and/or incorrect statement may be penalized.

1. (5) Perform indicated operations: $(\sqrt{4} + \sqrt{-9}) - (\sqrt{16} - \sqrt{-25}) + (\sqrt{36} - \sqrt{-49})$.

$$= (2 + 3i) - (4 - 5i) + (6 - 7i)$$

$$= 2 + 3i - 4 + 5i + 6 - 7i$$

$$= 4 + i //$$

2. (5) Perform indicated operations: $(8 + 9i)(10 - 11i)$.

$$= 80 - 88i + 90i - 99i^2$$

$$= 80 + 2i - 99(-1)$$

$$= 80 + 2i + 99$$

$$= 179 + 2i //$$

3. (5) Perform indicated operations and write in standard form: $\frac{12 + 13i}{\sqrt{196} - \sqrt{-225}}$.

$$= \frac{12 + 13i}{14 - 15i}$$

$$= \frac{12 + 13i}{14 - 15i} \cdot \frac{14 + 15i}{14 + 15i}$$

$$= \frac{168 + 180i + 182i + 195i^2}{196 - 225i^2}$$

$$= \frac{168 + 362i - 195}{196 + 225}$$

$$= \frac{-27 + 362i}{421}$$

$$= -\frac{27}{421} + \frac{362}{421}i //$$

4. (5) Perform indicated operations: $2\sqrt[3]{5x^7} + 4x\sqrt[3]{40x^4} - x\sqrt[3]{135}$.

$$\begin{aligned}
 &= 2 \cdot x^2 \sqrt[3]{5x} + 4x \cdot 2x \sqrt[3]{5x} - x \cdot 3 \sqrt[3]{5} \\
 &= 2x^2 \sqrt[3]{5x} + 8x^2 \sqrt[3]{5x} - 3x \sqrt[3]{5} \\
 &= 10x^2 \sqrt[3]{5x} - 3x \sqrt[3]{5} //
 \end{aligned}$$

5. (5) Rationalize the denominator: $\frac{\sqrt{3}-4\sqrt{2}}{2\sqrt{3}+5\sqrt{2}}$.

$$\begin{aligned}
 &= \frac{\sqrt{3}-4\sqrt{2}}{2\sqrt{3}+5\sqrt{2}} \cdot \frac{2\sqrt{3}-5\sqrt{2}}{2\sqrt{3}-5\sqrt{2}} \\
 &= \frac{2 \cdot 3 - 5\sqrt{6} - 8\sqrt{6} + 20 \cdot 2}{(2\sqrt{3})^2 - (5\sqrt{2})^2} \\
 &= \frac{6 - 13\sqrt{6} + 40}{4 \cdot 3 - 25 \cdot 2} \\
 &= \frac{46 - 13\sqrt{6}}{-38} // \\
 &= -\frac{46}{-38} - \frac{13\sqrt{6}}{-38} \\
 &= -\frac{23}{19} + \frac{13\sqrt{6}}{38} //
 \end{aligned}$$

6. (5) Solve: $\sqrt{1-4x}-5=x$.

$$\begin{aligned}
 \sqrt{1-4x} &= x+5 \\
 (\sqrt{1-4x})^2 &= (x+5)^2 \\
 1-4x &= x^2+10x+25 \\
 0 &= x^2+14x+24 \\
 0 &= (x+12)(x+2)
 \end{aligned}$$

$$\begin{array}{l|l}
 x+12=0 & x+2=0 \\
 \cancel{x} \neq -12 & x=-2 \\
 \{ -2 \} &
 \end{array}$$

7. (8) Solve: $\sqrt{3w+4} - \sqrt{w} = 2$.

$$\sqrt{3w+4} = 2 + \sqrt{w}$$

$$(\sqrt{3w+4})^2 = (2 + \sqrt{w})^2$$

$$3w + 4 = 4 + 4\sqrt{w} + w$$

$$2w = 4\sqrt{w}$$

$$(2w)^2 = (4\sqrt{w})^2$$

$$4w^2 = 16w$$

$$4w^2 - 16w = 0$$

$$4w(w - 4) = 0$$

$$4w = 0 \quad w - 4 = 0$$

$$w = 0 \quad w = 4$$

$$\{0, 4\} //$$

8. (8) Perform indicated operations and simplify:

$$\frac{q^2 + 6q}{6q + 12} \cdot \frac{q^2 - 5q}{2q + 10} \div \frac{q^2 + q - 30}{4q + 8}$$

$$= \frac{q^2 + 6q}{6q + 12} \cdot \frac{q^2 - 5q}{2q + 10} \cdot \frac{4q + 8}{q^2 + q - 30}$$

$$= \frac{q(q+6)}{\cancel{6}(q+2)} \cdot \frac{q(q-5)}{\cancel{2}(q+5)} \cdot \frac{\cancel{4}(q+2)}{(q+6)(q-5)}$$

$$= \frac{q^2}{3(q+5)} //$$

9. (8) Perform indicated operations and simplify: $\frac{m+n}{m+3n} - \frac{m-4n}{m-7n} + \frac{7mn+n^2}{m^2-4mn-21n^2}$.

$$\begin{aligned}
 &= \frac{m+n}{m+3n} - \frac{m-4n}{m-7n} + \frac{7mn+n^2}{(m-7n)(m+3n)} \\
 &= \frac{(m+n)(m-7n) - (m-4n)(m+3n) + 7mn+n^2}{(m-7n)(m+3n)} \\
 &= \frac{m^2-6mn-7n^2 - (m^2-mn-12n^2) + 7mn+n^2}{(m-7n)(m+3n)} \\
 &= \frac{m^2-6mn-7n^2 - m^2 + mn + 12n^2 + 7mn + n^2}{(m-7n)(m+3n)} \\
 &= \frac{-2mn + 6n^2}{(m-7n)(m+3n)} \\
 &= \frac{2n(m+3n)}{(m-7n)(m+3n)} = \frac{2n}{m-7n} //
 \end{aligned}$$

10. (8) Solve: $\frac{2}{a+3} - \frac{4}{a^2-4} = \frac{a+1}{a^2+5a+6}$.

$$\frac{2}{a+3} - \frac{4}{(a-2)(a+2)} = \frac{a+1}{(a+3)(a+2)}$$

$$(a-2)(a+2)(a+3) \left(\frac{2}{a+3} - \frac{4}{(a-2)(a+2)} \right) = \left(\frac{a+1}{(a+3)(a+2)} \right) (a-2)(a+2)(a+3)$$

$$2(a-2)(a+2) - 4(a+3) = (a+1)(a-2)$$

$$2(a^2-4) - 4a - 12 = a^2 - a - 2$$

$$2a^2 - 8 - 4a - 12 = a^2 - a - 2$$

$$a^2 - 3a - 18 = 0$$

$$(a-6)(a+3) = 0$$

$$\begin{array}{l|l}
 a-6=0 & a+3=0 \\
 a=6 & a \neq -3
 \end{array}$$

LCD = $(a-2)(a+2)(a+3)$; $a \neq -2, -3, 2$

{6} //

11. (8) Solve and express the solution set in interval

notation: $\frac{4}{c-2} < \frac{3}{c}$

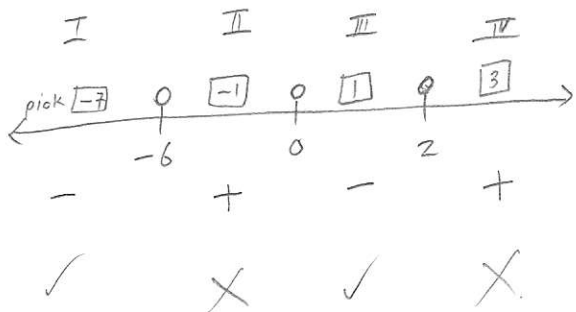
$$\frac{4}{c-2} - \frac{3}{c} < 0$$

$$\frac{4c - 3(c-2)}{c(c-2)} < 0$$

$$\frac{4c - 3c + 6}{c(c-2)} < 0$$

$$\frac{c + 6}{c(c-2)} < 0$$

| | |
|-------------|---------------------|
| <u>zero</u> | <u>undefined</u> |
| $c + 6 = 0$ | $c(c-2) = 0$ |
| $c = -6$ | $c = 0$ $c - 2 = 0$ |
| | $c = 2$ |



$(-\infty, -6) \cup (0, 2)$

12. (8) Simplify:

$$\frac{\left(4x^{\frac{1}{2}}y^{-\frac{1}{3}}\right)\left(x^6y^{-9}\right)^{\frac{1}{3}}}{\left(27x^{-4}y^{\frac{1}{2}}\right)^{\frac{1}{3}}}$$

$$= \frac{4x^{\frac{1}{2}}y^{-\frac{1}{3}} \cdot x^2y^{-3}}{27^{\frac{1}{3}}x^{-\frac{4}{3}}y^{-\frac{1}{6}}}$$

$$= \frac{4x^{\frac{5}{2}}y^{-\frac{10}{3}}}{3x^{-\frac{4}{3}}y^{-\frac{1}{6}}}$$

$$= \frac{4x^{\frac{5}{2} - (-\frac{4}{3})}y^{-\frac{10}{3} - (-\frac{1}{6})}}{3}$$

$$= \frac{4x^{\frac{23}{6}}y^{-\frac{19}{6}}}{3}$$

$$= \frac{4x^{\frac{23}{6}}}{3y^{\frac{19}{6}}}$$

side

$$x^{\frac{1}{2} + 2}$$

$$x^{\frac{5}{2}}$$

$$y^{-\frac{1}{3} + (-3)}$$

$$y^{-\frac{10}{3}}$$

$$x^{\frac{5}{2} + \frac{4}{3}}$$

$$x^{\frac{15+8}{6}}$$

$$y^{-\frac{10}{3} + \frac{1}{6}}$$

$$y^{-\frac{20+1}{6}}$$