Math 7A Test 1 (Ch 1 & 2) Name: 106

Show all necessary steps clearly, neatly, and systematically to receive full credit.

1. Find all solution(s): $5x^3 + 45x = 2x^2 + 18$.

$5x^3 - 2x^2 + 45x - 18 = 0$

$x^2(5x-2) + 9(5x-2) = 0$

$(5x-2)(x^2+9) = 0$

$5x-2=0$ $x^2+9=0$

$x = \frac{2}{5}$ $x^2 = -9$

$x = \pm\sqrt{-9}$

$x = \pm 3i$

$\{-3i, 3i, \frac{2}{5}\}$ //

2. Simplify: $\sqrt[3]{54x^7y^3} - x\sqrt[3]{-128x^4y^3} - x^2\sqrt[3]{-2xy^3}$.

27 2 -64 2 -1 2

$= 3x^2y\sqrt[3]{2x} - x\cdot -4xy\sqrt[3]{2x} - x^2\cdot -1y\sqrt[3]{2x}$

$= 3x^2y\sqrt[3]{2x} + 4x^2y\sqrt[3]{2x} + x^2y\sqrt[3]{2x}$

$= 8x^2y\sqrt[3]{2x}$ //

3. Solve and write the solution set in interval notation: $-|-11-7x|+2<-10$.

$-|-11-7x| < -12$

$|-11-7x| > 12$

$-11-7x > 12$ or $-11-7x < -12$

$-7x > 23$ $-7x < -1$

$x < \frac{23}{-7}$ $x > \frac{1}{7}$

$(-\infty, -\frac{23}{7}) \cup (\frac{1}{7}, \infty)$ //

l_1 l_2

4. Find the equation of the line passes through (- 4 , 3) and perpendicular to $3x - 5y = 7$. Write the result in standard form.

$l_1 \perp l_2$

$(-4, 3)$, $m = -\frac{5}{3}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{5}{3}(x - (-4))$

$y - 3 = -\frac{5}{3}x - \frac{20}{3}$

$\frac{5}{3}x + y = -\frac{20}{3} + 3$

$5x + 3y = -20 + 9$

$5x + 3y = -11$ /

$3x - 5y = 7$

$-5y = -3x + 7$

$y = \frac{-3}{-5}x + \frac{7}{-5}$

$y = \frac{3}{5}x - \frac{7}{5}$

$m = \frac{3}{5}$

5. Solve: $\sqrt{3x+7} + \sqrt{x+2} = 1$.

$\sqrt{3x+7} = 1 - \sqrt{x+2}$

$(\sqrt{3x+7})^2 = (1 - \sqrt{x+2})^2$

$3x + 7 = 1 - 2\sqrt{x+2} + x + 2$

$3x + 7 = x + 3 - 2\sqrt{x+2}$

$2x + 4 = -2\sqrt{x+2}$

$-x - 2 = \sqrt{x+2}$

$(-x-2)^2 = (\sqrt{x+2})^2$

$x^2 + 4x + 4 = x + 2$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x + 2 = 0$

$x = -2$

$x + 1 = 0$

$\cancel{x = -1}$

$\{-2\}$ //

6. Simplify and write in standard form: $\frac{-2i}{(3-i)^2}$.

$$= \frac{-2i}{9-6i+i^2}$$

$$= \frac{-2i}{9-6i-1}$$

$$= \frac{-2i}{8-6i}$$

$$= \frac{-2i}{2(4-3i)}$$

$$= \frac{-i}{4-3i}$$

$$= \frac{-i}{4-3i} \cdot \frac{4+3i}{4+3i}$$

$$= \frac{-4i-3i^2}{16-9i^2}$$

$$= \frac{3-4i}{25}$$

$$= \frac{3}{25} - \frac{4}{25}i \;//$$

7. Solve: $\frac{-3}{x+4}+\frac{7}{x-4}=\frac{-5x+4}{x^2-16}$.

$\frac{-3}{x+4}+\frac{7}{x-4}=\frac{-5x+4}{(x+4)(x-4)}$ $\quad x \neq 4, -4$

$$-3(x-4)+7(x+4) = -5x+4$$

$$-3x+12+7x+28 = -5x+4$$

$$4x+40 = -5x+4$$

$$9x = -36$$

$$x = -4$$

$$\phi$$

8. Test for symmetry: $y^2 = \frac{\sqrt[3]{x}}{x^3}$.

x-axis

$(-y)^2 = \frac{\sqrt[3]{x}}{x^3}$

$y^2 = \frac{\sqrt[3]{x}}{x^3}$

symmetry about x-axis.

y-axis

$y^2 = \frac{\sqrt[3]{-x}}{(-x)^3}$

$y^2 = \frac{-\sqrt[3]{x}}{-x^3}$

$y^2 = \frac{\sqrt[3]{x}}{x^3}$

symmetry about y-axis.

origin

$(-y)^2 = \frac{\sqrt[3]{-x}}{(-x)^3}$

$y^2 = \frac{-\sqrt[3]{x}}{-x^3}$

$y^2 = \frac{\sqrt[3]{x}}{x^3}$

symmetory about origin.

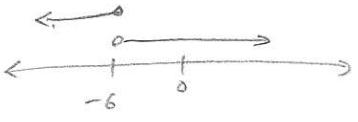
9. Solve and write the solution set in set-builder notation: $-\frac{7}{3}x < 14$ and $-3x + 2 \geq 20$.

intersection.

$x > 14 \cdot -\frac{3}{7}$ and $-3x \geq 18$

$x > -6$ $x \leq -6$

Side



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10. Find center and radius of the circle: $x^2 + y^2 + 4x - 20y + 95 = 0$.

$$X^2 + 4x + y^2 - 20y = -95$$

$$x^2 + 4X + 4 + y^2 - 20y + 100 = -95 + 4 + 100$$

$$(x+2)^2 + (y-10)^2 = 9$$

center $= (-2, 10)$

radius $= 3$

11. The relationship between Celsius C and Fahrenheit F degrees of measuring temperature is linear. Find a linear equation relation C and F if $0^\circ C$ corresponds to $32^\circ F$ and $100^\circ C$ corresponds to $212^\circ F$. Use the equation to find the Celsius measure of $70^\circ F$.

$$(C, F) \rightarrow (\overset{x_1}{0}, \overset{y_1}{32}) \; \& \; (100, 212)$$

$$y = mx + b$$

$$F = \frac{9}{5} C + 32 \; //$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{212 - 32}{100 - 0}$$

$$= \frac{180}{100}$$

$$= \frac{9}{5}$$

$$b = (0, 32)$$

$$70^\circ F \rightarrow C^\circ = ?$$

$$70 = \frac{9}{5} C + 32$$

$$38 = \frac{9}{5} C$$

$$38 \cdot \frac{5}{9} = C$$

$$\frac{190}{9} = C \; //$$

12. Solve for D: $A = B\cdot\sqrt[3]{\frac{C}{D}} - E$.

$$A + E = B \cdot \sqrt[3]{\frac{C}{D}}$$

$$\frac{A+E}{B} = \sqrt[3]{\frac{C}{D}}$$

$$\left(\frac{A+E}{B}\right)^3 = \frac{C}{D}$$

$$\frac{(A+E)^3}{B^3} = \frac{C}{D}$$

$$\frac{B^3}{(A+E)^3} = \frac{D}{C}$$

$$\frac{C\,B^3}{(A+E)^3} = D \quad //$$

x_1 y_1

13. Consider A(1 , 4) and B(-3 , 2) are the endpoints of a diameter of a circle.

a. Find the slope of the segment AB.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{-3-1} = \frac{-2}{-4} = \frac{1}{2} \quad //$$

b. Find the center of the circle.

Center = midpoint of AB $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{1+(-3)}{2}, \frac{4+2}{2}\right) = (-1, 3)$ //

c. Find the radius of the circle.

$$r = \frac{d}{2} = \frac{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}{2} = \frac{\sqrt{(-4)^2+(-2)^2}}{2} = \frac{\sqrt{20}}{2} = \sqrt{5} \quad //$$

d. Write equation of the circle in standard form.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x+1)^2 + (y-3)^2 = 5 \quad //$$

e. Write the equation of the tangent line to the circle at the point (1 , 4). Write the result in slope-intercept form.

↑ and radius are perpendicular.

$m = -2$

$(1, 4)$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -2(x-1)$$

$$y - 4 = -2x + 2$$

$$y = -2x + 6 \quad //.$$